

# Correcting the Foundations of Operations Research

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# References




[ScientificMetrics.com/publications.html](http://ScientificMetrics.com/publications.html)

- [1] Jonathan Barzilai, “Game Theory Foundational Errors” Parts I–VI.
- [2] Jonathan Barzilai, “Value of a Game,” Notices of the American Mathematical Society, Vol. 55, No. 4, p. 446, April 2008.
- [3] Jonathan Barzilai, “On the Mathematical Foundations of Economic Theory,” pp. 1–13, 2007.


# Need for Corrections (1)

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- The chapters on decision theory and game theory in Hillier & Lieberman are fundamentally wrong.
  - Chapters 13, 14, 15 in Winston contain fundamental errors.
  - Decision theory is based on preference measurement. Measurement theory is founded on fundamental errors.

# Need for Corrections (2)

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- Decision Analysis Society – no refutation, yet no correction.
  - Howard/Edwards “In Praise of the Old Time Religion”
  - Prescribing the conclusions when the assumptions are not satisfied.

# Need for Corrections (3)


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- The leaders of INFORMS's Decision Analysis Society have been aware of foundational errors since at least 2005.
  - Nevertheless, they have taken no action to advise their followers and correct these errors.
  - Decision analysis that is based on flawed methodologies (including utility theory) produces meaningless numbers.

# “Foundations of Measurement” has no foundations



- By David H. Krantz, R. Duncan Luce, Patrick Suppes, Amos Tversky, vol. 1 1971, vol 2 1989, vol 3 1990; >1300 pages.
- No definition of “unit”!
- Incorrect models for the most elementary variables – length and weight. (Luce 1992: “Everybody involved in this research has been aware all along that the class of homogeneous structures fails to include a number of scientifically important examples of classical physical measurement and, quite possibly, some that are relevant to the behavioral sciences.”)

# Questions

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- On a Scale of 0-10 ... ?
  - Have you seen a proof of the applicability of addition and multiplication on scale values for any scale?
  - Whose values?
  - What's wrong with  
 $2008+2009 = 4017?$


# The Fundamental Issue



Conditions for applicability of mathematical operations:

- There must be conditions.
- Not in literature.
- (For non-physical variables.)
  
- Inapplicable operations produce meaningless numbers.

# Von Neumann and Morgenstern's Errors

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- Decision theory, game theory, and other theories are founded on errors committed by von Neumann and Morgenstern.
  - Addition and multiplication are not applicable on utility scale values.
  - Additional Game Theory Errors: see “Game Theory Foundational Errors” – Parts I-VI.

# Is there a Basis for the Application of Mathematics in the Social Sciences?



Campbell 1920-1940: No!

Von Neumann and Morgenstern:  
Yes! Game theory as the mathe-  
matical basis for economics and  
utility as the basis for game theory.

A difficult problem; many errors.

# A Sample of Game Theory Errors

- Undefined sums:

$$2008+2009=4017.$$

- Values of coalitions: whose values?

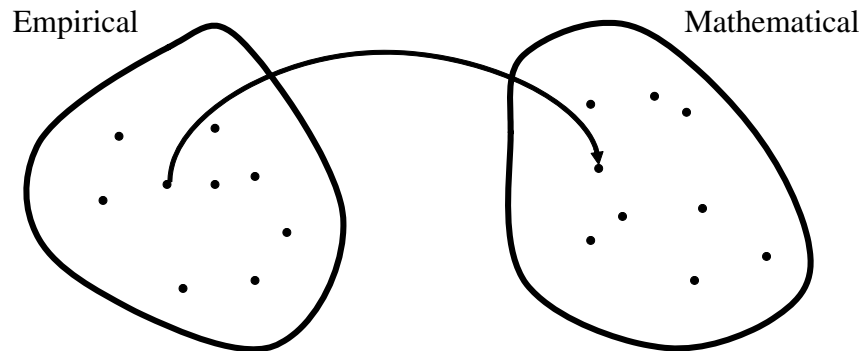
- The two-person “zero-sum” game

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \text{ with “payoff” table } \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} :$$

Choose  $[C, D]$  with probability 0.25 regardless of what  $D$  is!

# The Modelling Framework

This Framework is *essential!*



- System: set(s) with operations.
- A scale is a mapping that reflects operations (homomorphism).
- Not just any arbitrary mapping.
- The system  $M$  is a model of  $E$ .
- Property: e.g. adding length.

# The Role of the *Property* in Measurement

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- The modelling framework is the only foundation for the mathematical operations.
  - We do not “add objects” - we add their property (length, mass, etc.)!
  - The property is part of the framework.
  - The property of interest here is preference.

# The Principle of Reflection




- Mathematical operations are enabled only if they are reflections of corresponding empirical operations.
- Note: order is not an operation.
- Implications: addition and multiplication are not applicable on scale values in the classical literature.

# Money is Not a Property of Objects

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- Examples of money (Shubik): “coconuts, cocoa beans, dried fish, salt bars, or a beaver pelt” – these are objects, not a property of objects.
- The outcome of measuring temperature is a temperature scale. The outcome of measuring mass is not a length scale.
- Von Neumann and Morgenstern measure preference and produce a utility scale!

# Better Understanding is Needed

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- Role of property under measurement.
  - Distinction between objects and properties of objects.
  - Preference, utility, value, etc., are all synonyms.
  - The “utility of value” of an object (Keeney & Raiffa) - the “length of the length” of a pen??!

# Applicability of Mathematical Operations, 1887-1940



1887, Helmholtz

1920, Campbell

1932-1940, British Association for the Advancement of Science

1940, Controversy not resolved in Final Report.

# Von Neumann and Morgenstern, 1944

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The modelling framework has never been in dispute.

Solved the wrong problem:

- Instead of addition and multiplication, axioms for “center of gravity” operation and  $t \Rightarrow p + qs$  scale uniqueness.
- But the problem was the empirical operations and  $t \Rightarrow p + qs$  does not imply addition and multiplication.

# Implications for Economic Theory

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- Addition and multiplication are not applicable on ordinal scale values.
- Differentiation is not applicable on ordinal utility.
- If the utility scales of consumer demand theory are ordinal they cannot be differentiated. If they are differentiable they are not ordinal.

# Utility Shortcomings



The Principle of Reflection:

- Addition and multiplication are two binary operations; “center of gravity” is one ternary operation.
- Addition and multiplication are not applicable on scale values for *any* scale in the literature including utility scales.

# Utility Shortcomings (cont.)




Barzilai's paradox – more important than common utility paradoxes. An intrinsic contradiction:

- Existence and uniqueness vs. construction.
- Framework ok in the abstract. For *preference*, the *interpretation* of the empirical operation leads to a contradiction – prizes unconstrained, lotteries constrained, prizes that are lotteries allowed.

And there is more.

# Game Theory

## Shortcomings



Game theory values: a fundamental concept. Assigning values to outcomes, coalitions, etc., is constructing value functions.

Value is not a physical variable. Whose values? What is  $v(S)$ ?

The characteristic function of a game and other central game theory concepts including imputations, Shapley's value, and vNM's solution are ill-defined!

# Game Theory

## Shortcomings (cont.)

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Undefined Sums:

$v(S)+v(T)$  is undefined for *utility* scales, *time*, *potential energy*, and similar variables. The sum of “imputations” is undefined. Von Neumann & Morgenstern’s solution is ill-defined.

The utility of a coalition:

Reduction to a two-person game – a coalition vs. its complement. But there is no basis for the utility of a group of players.

# Game Theory

## Shortcomings (cont.)




“The” value of a game:

The value of a two-person zero-sum game is ill-defined: Utility scales are not unique,  $t = p + qs$ . Varying  $p$  or  $q$  changes the value. Any number can be the value of the game.

And there is more.

# Back to 1940



Von Neumann and Morgenstern's utility theory and game theory cannot serve as a foundation.

(Note: the notion that the only type of utility that is needed in economic theory is ordinal, is an error.)

# The 1940 Final Report



- Missed multiplication.
- Missed the form of addition and multiplication for *time*, *potential energy*, etc.
- Incorrect model even for *mass* and *length*.
- No correction in the literature.

# Homogeneous Fields (1)




The problem does have a solution.

The mathematical system  $M$  is a field.

The empirical system  $E$  is homogeneous.

We need to construct a homogeneous field.

# Homogeneous Fields (2)

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- A field: one set, two operations including inverse operations. Absolute zero, absolute one.
  - A vector space: 2 sets (vectors and scalars), mixed multiplication. Absolute zero.
  - A one-dimensional vector space. *Length* and *mass*.
  - An affine space: 3 sets (points, vectors, scalars), form of operations.

# Homogeneous Fields (3)



- A new classification: weak, proper, and strong scales. Field (scalar), vector, and affine scales.
- Constructing strong affine scales.

# Post-vNM

## “Measurement Theory”

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- The issue is mathematical modeling of measurement, applicability of operations.
- Uniqueness classification: applicability of operations disappears from the literature.
- Addition and multiplication not applicable on scale values for *any* scale in the literature.

# “Measurement Theory” (2)




- Model for *length*: addition without multiplication (multiplication is not repeated addition).
- “Extensive” measurement & ratio scales: half the operations are lost; derived from *position*.
- “Difference” measurement & interval scales: the other half is lost; neither addition nor multiplication are applicable.

# Ordinal Utility & Indifference Curves

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- Ordinal scale values cannot be differentiated.
  - Differentiable scales are not ordinal.
  - See paper by this title.

# AHP

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- Problems mis-diagnosed. Rank reversal is not the main issue. (Dyer's analysis is circular and of no value.)
  - Addition, multiplication, matrix, and vector operations are not applicable; eigenvector is the wrong solution; preference ratios are undefined.
  - Additional errors.