

Foundational Errors in Game Theory, Economic Theory, and Other Social Sciences

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References



ScientificMetrics.com/publications.html

- [1] Jonathan Barzilai, *Game Theory Foundational Errors Parts I–IV*.
- [2] J. Barzilai, *On Ordinal, Cardinal, and Expected Utility*, pp. 1–6, 2011. Posted at www.ScientificMetrics.com
- [3] Jonathan Barzilai, “Preference Function Modeling: The Mathematical Foundations of Decision Theory,” in *Trends in Multiple Criteria Decision Analysis*, Matthias Ehrgott, José Rui Figueira, Salvatore Greco (Eds.), Springer, pp. 57–86, 2010. Preprint at www.ScientificMetrics.com

The Ordinal Utility Error (1)




- It is claimed in microeconomic theory that *ordinal* utility is sufficient to derive differential optimality conditions.
- The concept of slope is undefined on ordinal topographic maps.
- Ordinal spaces are not vector spaces. Addition, multiplication, and differentiation are not applicable on ordinal scale values.

The Ordinal Utility Error (2)




- Errors committed by Pareto, Hicks, Samuelson, and Debreu are still in the literature.
- If microeconomics' utility scales are ordinal they are not differentiable. If they are differentiable they are not ordinal.


“Cardinal” Utility

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- If it is not ordinal it is cardinal (e.g. Mas-Colell et al.) -- is not a proper definition.
 - Cardinal time? Cardinal potential energy?
 - Intensity of preference -- intensity of time?
 - Strength of preference -- strength of time?


Vector Space Operations

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- n -tuples vs. vectors.
 - Vector space operations are not applicable in ordinal or affine spaces.
 - Inapplicable vector-space operations define game theory's characteristic function.

Von Neumann and Morgenstern's Errors (1)

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- Errors committed by von Neumann and Morgenstern are at the foundations of Game Theory, Economic Theory, Decision Theory, and related theories.
 - Decision theory is based on preference measurement. Measurement theory is founded on fundamental errors.

Von Neumann and Morgenstern's Errors (2)

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- Addition and multiplication are not applicable on utility scale values.
 - Additional Game Theory Errors: see *Game Theory Foundational Errors Parts I-IV*.

The Characteristic Function of Game Theory



$$v(S)+v(T):$$


Whose values?

Undefined sums.

vNM's solution of a game, imputations, Shapley's value -- based on errors.

Zero-sum two-person game theory is formulated incorrectly.

Applicability of Operations vs. Scale Uniqueness – Solving the Wrong Problem

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- Campbell & Ferguson et al. (1940).
 - The purpose of measurement.
 - vNM's goal. Solving the wrong problem and proving the wrong theorem.

The Fundamental Issue

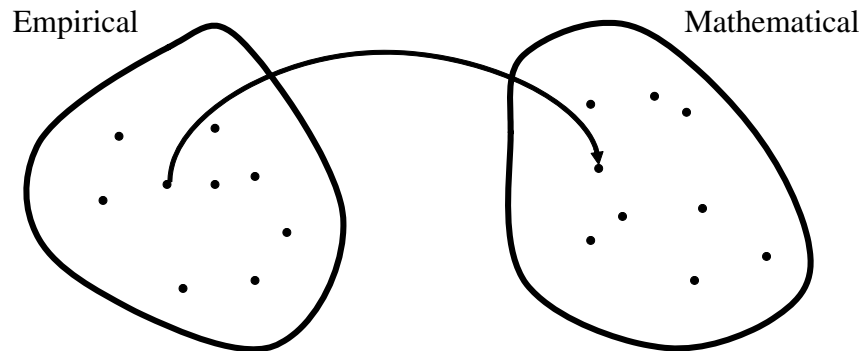


Conditions for applicability of mathematical operations:

- There must be conditions.
- Not in literature.
- For non-physical variables.
- Inapplicable operations produce meaningless results.

The Modelling Framework

This Framework is *essential!*



- System: set(s) with operations.
- A scale is a mapping that reflects operations (homomorphism).
- Not just any arbitrary mapping.
- The system M is a model of E .
- Property: e.g. adding length.

The Role of the *Property* in Measurement

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- The modelling framework is the only foundation for the mathematical operations.
 - We do not “add objects” - we add their property (length, mass, etc.)!
 - The property is part of the framework.
 - The property of interest here is preference.

The Principle of Reflection




- Mathematical operations are enabled only if they are reflections of corresponding empirical operations.
- Note: order is not an operation.
- Implications: addition and multiplication are not applicable on scale values in the classical literature.

Money is Not a Property of Objects

- Examples of money (Shubik): “coconuts, cocoa beans, dried fish, salt bars, or a beaver pelt” – these are objects, not a property of objects.
- The outcome of measuring temperature is a temperature scale. The outcome of measuring mass is not a length scale.
- Von Neumann and Morgenstern measure preference and produce a utility scale!

Better Understanding is Needed

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- Role of property under measurement.
 - Distinction between objects and properties of objects.
 - Preference, utility, value, etc., are all synonyms.
 - The “utility of value” of an object (Keeney & Raiffa) - the “length of the length” of a pen??!

Applicability of Mathematical Operations, 1887-1940



1887, Helmholtz

1920, Campbell

1932-1940, British Association for the Advancement of Science

1940, Controversy not resolved in Final Report.

Von Neumann and Morgenstern, 1944

The modelling framework has never been in dispute.

Solved the wrong problem:

- Instead of addition and multiplication, axioms for “center of gravity” operation and $t \Rightarrow p + qs$ scale uniqueness.
- But the problem was the empirical operations and $t \Rightarrow p + qs$ does not imply addition and multiplication.