Correcting Foundational Errors in Decision Theory, Game Theory, and Other Disciplines

Jonathan Barzilai

ScientificMetrics.com/publications.html

Canadian Op. Res. Society

Annual Conference

Niagara Falls, Ontario

11 June 2012

References

ScientificMetrics.com/publications.html

[1] Jonathan Barzilai, "Game Theory Foundational Errors" Parts I–IV.

[2] Jonathan Barzilai, "Preference Function Modeling: The Mathematical Foundations of Decision Theory," in *Trends in Multiple Criteria Decision Analysis*, Matthias Ehrgott, José Rui Figueira, Salvatore Greco (Eds.), Springer, pp. 57–86, 2010. Preprint at www.ScientificMetrics.com

Von Neumann and Morgenstern's Errors (1)

- Errors committed by von Neumann and Morgenstern are at the foundations of Game Theory, Economic Theory, Decision Theory, and related theories.

- Decision theory is based on preference measurement. Measurement theory is founded on fundamental errors.

Von Neumann and Morgenstern's Errors (2)

- Addition and multiplication are not applicable on utility scale values.

- Additional Game Theory Errors: see "Game Theory Foundational Errors" – Parts I-IV.

Economic Theory Errors (1)

━━━━━━━━━━━━━━━━━━

- A minor error by Pareto (1906) magnified by Hicks (~1935), Samuelson (1948), Debreu (1954).

- Hicks:

  "Pure economics has a remarkable way of producing rabbits out of a hat" ... "It is fascinating to try to discover how the rabbits got in; for those of us who do not believe in magic must be convinced that they got in somehow."

Economic Theory Errors (2)

- Advanced differential calculus without addition and multiplication?

- Hicks's rabbits are still in the literature; Mas-Colell, and many others.

- Group decision making: Arrow's non-existence theorem: ordinal, not constructive.

The Ordinal Utility Error

- Although addition and multiplication are not applicable on ordinal utility values, it is claimed in the economic theory literature that ordinal utility is sufficient to derive differential optimality conditions.

- This is an error. The conditions for applying calculus theorems are not satisfied.

"Cardinal" Utility (!?)

- If it is not ordinal it is cardinal (!?)

- Cardinal time? Cardinal potential energy?

Non-linear Preference (!?)

- Non-linear potential energy?

- Non-linear temperature?

Strength of Preference (!?)

- Strength of time?

- Strength of energy?

Questions

- On a Scale of 0-10 ... ?

- What's wrong with
  $$2011+2012 = 4023?$$

A Sample of

Game Theory Errors

- Undefined sums
  (same as in 2011+2012=4023).

- Values of coalitions: whose values?

- The two-person "zero-sum" game $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ with "payoff" table $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$: Choose [C, D] with probability 0.25 regardless of what D is! (Part V)

Multiplicative Weight (!?)

The "theory of measurement" does not distinguish between additive and multiplicative models.

•Concatenating rods of lengths 6 and 7 yields 42!?

Any Space is a Vector Space (!?)

- Any *n*-tuple is a vector and any space is Euclidean!?

- Vector space operations are applicable whether they are or are not!? (as for example in the AHP)

The Utility of Value (!?)

- The length of my weight?

- Without the property under mea-surement we cannot introduce math to science.

- Money is not a property of objects. Game theory depends on utility theory: outcomes vs. payoff (losing a leg).

The Two Bottom Lines of an All-Star Cast of Utility Theorists

- (1) They unanimously agreed that subjective expected utility is the appropriate normative rule for decision making under risk or uncertainty.

- (2) They all agreed that the experimental and observational evidence has established as a fact the assertion that people do not maximize subjective expected utility.

Applicability of Operations vs. Scale Uniqueness – Solving the Wrong Problem

<hr>

- Campbell & Ferguson et al. (1940).

- The purpose of measurement.

- vNM's goal. Solving the wrong problem and proving the wrong theorem.

The Fundamental Issue

- There must be conditions for applicability of mathematical operations. Not in literature.

- (For non-physical variables.)

- Which operations are applicable in which mathematical space? (see the paper "On ordinal, Cardinal, and Expected Utility")

- Inapplicable operations produce meaningless numbers.

The Characteristic Function of Game Theory
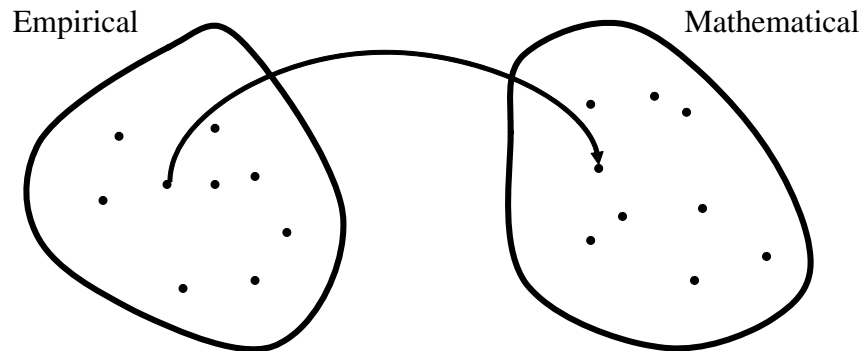
---

v(S)+v(T):

Whose values?

Undefined sums.

vNM's solution of a game, imputa-tions, Shapley's value -- based on errors.

Zero-sum two-person game theory is formulated incorrectly.

The Modelling Framework

This Framework is *essential!*

Empirical   Mathematical

- System: set(s) with operations.

- A scale is a mapping that reflects operations (homomorphism).

- Not just any arbitrary mapping.

- The system *M* is a model of *E.*

- Property: e.g. adding length.

The Role of the *Property* in Measurement

- The modelling framework is the only foundation for the mathematical operations.

- We do not "add objects" - we add their property (length, mass, etc.)!

- The property is part of the framework.

- The property of interest here is preference.

The Principle of Reflection

- Mathematical operations are enabled only if they are reflections of corresponding empirical operations.

- Note: order is not an operation.

- Implications: addition and multiplication are not applicable on scale values in the classical literature.

Money is Not
a Property of Objects

- Examples of money (Shubik): "coconuts, cocoa beans, dried fish, salt bars, or a beaver pelt" – these are objects, not a property of objects.

- The outcome of measuring temperature is a temperature scale. The outcome of measuring mass is not a length scale.

- Von Neumann and Morgenstern measure preference and produce a utility scale!

Homogeneous Fields (1)

The problem does have a solution.

The mathematical system M is a field.

The empirical system E is homogeneous.

We need to construct a homogeneous field.

Homogeneous Fields (2)

- A field: one set, two operations including inverse operations. Absolute zero, absolute one.

- A vector space: 2 sets (vectors and scalars), mixed multiplication. Absolute zero.

- A one-dimensional vector space. *Length* and *mass*.

- An affine space: 3 sets (points, vectors, scalars), form of operations.

Homogeneous Fields (3)

- A new classification: weak, proper, and strong scales. Field (scalar), vector, and affine scales.

- Constructing strong affine scales.

Better Understanding
is Needed

- Role of property under measure-ment.

- Distinction between objects and properties of objects.

- Preference, utility, value, etc., are all synonyms.

- The "utility of value" of an object (Keeney & Raiffa) - the "length of the length" of a pen??!

Applicability of Mathematical Operations, 1887-1940

1887, Helmholtz

1920, Campbell

1932-1940, British Association for the Advancement of Science

1940, Controversy not resolved in Final Report.

Von Neumann and Morgenstern, 1944

The modelling framework has never been in dispute.

Solved the wrong problem:

- Instead of addition and multiplication, axioms for "center of gravity" operation and $t=p+qs$ scale uniqueness.

- But the problem was the empirical operations and $t=p+qs$ does not imply addition and multiplication.

Implications for Economic Theory

- Addition and multiplication are not applicable on ordinal scale values.

- Differentiation is not applicable on ordinal utility.

- If the utility scales of consumer demand theory are ordinal they cannot be differentiated. If they are differentiable they are not ordinal.

Utility Shortcomings (1)

The Principle of Reflection:

- Addition and multiplication are two binary operations; "center of gravity" is one ternary operation.

- Addition and multiplication are not applicable on scale values for *any* scale in the literature including utility scales.

Utility Shortcomings (2)

Barzilai's paradox – more important than common utility paradoxes. An intrinsic contradiction:

- Existence and uniqueness vs. construction.

- Framework ok in the abstract. For *preference*, the *interpretation* of the empirical operation leads to a contradiction – prizes unconstrained, lotteries constrained, prizes that are lotteries allowed.

And there is more.

Game Theory Shortcomings (1)

Game theory values: a fundamental concept. Assigning values to outcomes, coalitions, etc., is constructing value functions.

Value is not a physical variable. Whose values? What is $v(S)$?

The characteristic function of a game and other central game theory concepts including imputations, Shapley's value, and vNM's solution are ill-defined!

Game Theory
Shortcomings (2)

Undefined Sums:

$v(S)+v(T)$ is undefined for *utility* scales, *time, potential energy,* and similar variables. The sum of "imputations" is undefined. Von Neumann & Morgenstern's solution is ill-defined.

The utility of a coalition:

Reduction to a two-person game – a coalition vs. its complement. But there is no basis for the utility of a group of players.

Game Theory
Shortcomings (3)

"The" value of a game:

The value of a two-person zero-sum game is ill-defined: Utility scales are not unique, $t=p+qs$. Varying p or q changes the value. Any number can be the value of the game.

And there is more.

Back to 1940

Von Neumann and Morgenstern's utility theory and game theory cannot serve as a foundation.

(Note: the notion that the only type of utility that is needed in economic theory is ordinal, is an error.)

The 1940 Final Report

- Missed multiplication.

- Missed the form of addition and multiplication for *time, potential energy,* etc.

- Incorrect model even for *mass* and *length.*

- No correction in the literature.

Post-vNM
"Measurement Theory"

- The issue is mathematical modelling of measurement, applicability of operations.

- Uniqueness classification: applicability of operations disappears from the literature.

- Addition and multiplication not applicable on scale values for *any* scale in the literature.

"Measurement Theory" (2)

- Model for *length*: addition without multiplication (multiplication is not repeated addition).

- "Extensive" measurement & ratio scales: half the operations are lost; derived from *position*.

- "Difference" measurement & interval scales: the other half is lost; neither addition nor multiplication are applicable.

Ordinal Utility & Indifference Curves

- Ordinal scale values cannot be differentiated.

- Differentiable scales are not ordinal.

- See paper by this title.

AHP (1)

- Problems mis-diagnosed. Rank reversal is not the main issue. (Dyer's analysis is circular and of no value.)

- Addition, multiplication, matrix, and vector operations are not applicable; eigenvector is the wrong solution; preference ratios are undefined.

AHP (2)

- Weighted sum cannot correspond to relative importance.

- Verbal scales.

- 1-9 values are arbitrary.

- Reference to Miller's work.

- Additional errors.