September 16, 2014

Professor William D. Nordhaus
President
American Economic Association

Dear Professor Nordhaus,

Mathematical economics is founded on errors that require correction as they affect the education of a large number of students. Here are a few examples:

Game Theory
Game theory, which was to be the basis for economic theory, is founded on errors. For examples see the *Notices of the American Mathematical Society* [1, 2008] and Barzilai [2, §3.6.8, 2010] where I show that even for game theory's most elementary model of a two-person zero-sum game, the theory's solution leads to irrational strategies.

The Ordinal Utility Error
In economic preference theory, the claim that ordinal functions can be differentiated is based on elementary errors: Theorems of differential calculus are applied where the assumptions of these theorems are not satisfied and mathematical operations are applied where they are not applicable. The notion that ordinal functions are differentiable is an error. It has no parallel in mathematics and science. For a detailed analysis of these and additional errors see [2].

Demand Theory
Demand quantities are determined in Marshallian theory by maximizing the consumer's utility under a budget constraint. In contrast, Hicksian demand quantities are determined by minimizing expenditure, subject to keeping the consumer's utility value fixed at its current value. The fact that these theories produce different quantities raises the question: Which quantities are the "correct" ones? This obvious question leads not to two contradictory demand theories, but to infinitely many ones: The consumer may wish to keep the utility value fixed at its current value $u(0)$, or maximize it to its greatest value $u(1)$ under the budget constraint, or raise it to an intermediate level $u(x)$ where x is any fraction between zero and one. (The consumer may spend the entire budget, none of it, or any fraction of it.) Each fraction x corresponds to a different *x-fraction demand theory* and these theories produce infinitely many different demand quantities. (The correct theoretical model is neither Marshallian nor Hicksian nor any *x-fraction* theory since all these theories ignore the consumer's preference for money. To determine the correct quantities the consumer's utility function must include money as one of the variables.)

Applicability of Mathematical Operations
Identifying the conditions of applicability of the operations of algebra and calculus is a foundations-of-science problem. The conditions for applicability of the operations of addition and multiplication are not considered in the literature and are violated throughout mathematical economics. Inapplicable operations have been applied in microeconomics; in particular, vector space operations are not applicable on ordinal data and ordinal functions are not differentiable. Physics and mathematics should be rewritten if ordinal information is sufficient for the application of differential calculus. Some of the errors are the result of applying theorems of differential calculus where the assumptions of these theorems are not satisfied. Applying theorems where the conditions for their applicability are not satisfied is an elementary error in consumer demand theory. The onus is on microeconomics authors to establish that the assumptions of theorems they use are satisfied. For details see [3 and 4].

Corrections Necessary
As an educator and a leader of the economics profession, I trust that you agree that errors in microeconomics must be corrected rather than defended to ensure that students are not taught flawed theories.

I look forward to your response.

Yours truly,

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Professor
Dalhousie University

References

[1]   J. Barzilai, "Value of a Game," *Notices of the American Mathematical Society,* Vol. 55 No. 4, p. 446, April 2008.
      http://www.ams.org/notices/200804/tx080400446p.pdf

[2]   Jonathan Barzilai, "Preference Function Modeling: The Mathematical Foundations of Decision Theory," in *Trends in Multiple Criteria Decision Analysis,* Matthias Ehrgott, José Rui Figueira, Salvatore Greco (Eds.), Springer, pp. 57–86, 2010.

[3]   Jonathan Barzilai, "Inapplicable Operations on Ordinal, Cardinal, and Expected Utility," *Real-World Economic Review*, No. 63, pp. 98–103, 25 March 2013.

[4]   Jonathan Barzilai, "Demand Theory is Founded on Errors," *Real-World Economic Review*, No. 68, pp. 62–65, 21 August 2014.