# Supply and Demand Equilibrium and Debreu's Theory of Value

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#### **Abstract**

The theory of supply and demand equilibrium, including Debreu's *Theory of Value*, is based on errors. Errors in preference theory, demand theory, and equilibrium theory are noted.

#### **Keywords**

Mathematical Economics, Theory of Value, Supply, Demand, Equilibrium

#### 1 Introduction

In addition to the ordinal utility error of Slutsky, Hicks, and Samuelson [16, 13, and 15], and other preference theory errors (see Barzilai [8, 10, and 12]), the conflicting Marshallian and Hicksian demand theories are both based on an error (see [3]) as is the use of demand and supply (and other) "curves" in economic theory (see [1]). Supply and demand equilibrium theory is also founded on errors, even in the case of a market for a single commodity, and theorems that specify conditions for existence of equilibrium leave important questions unanswered.

#### 2 Producers vs. Consumers

Producers and consumers are decision makers, and in the context of this theory the distinction between them is a source of error. Producers maximize their preferences as do consumers, but in Debreu's theory producers maximize profits rather than preferences. What producers maximize is, of course, their choice rather than Debreu's. That may or may not be profit. It may be market share rather than profit, or it may be

another variable, but a theory that takes the choice away from decision makers is not a theory of choice. In a "Robinson Crusoe economy" – a one-man economy – Robinson the consumer is entitled to his preferences but Robinson the producer is not, according to this self-contradictory theory.

## 3 Preferences

The mathematical space in which the operations of addition and multiplication are applicable on preference, utility, or value scales has not been correctly identified in the literature of decision theory and economics, and inapplicable operations have been applied on preference scale values throughout the literature. For details see Barzilai [4, 5, 8, 10, and 12]. In particular, the operations of algebra and differential calculus are not applicable on ordinal utility scale values and the ordinal utility error of Slutsky, Hicks, and Samuelson (see [5]) is not corrected in Debreu's "Theory of Value." In addition, as noted above, a correct theory of preference does not take the choice away from decision makers.

# 4 Supply and Demand Equilibrium

A commodity's price depends on its supply and demand quantities. According to economic theory, equilibrium is achieved when the supply quantity  $Q_s$  equals the demand quantity  $Q_d$ . In essence, equilibrium theory states that under certain conditions there exists a price (prices in the case of multiple commodities) for which supply and demand quantities are equal:  $Q_s - Q_d = 0$ .

Consider a market for a single commodity for which at a given time  $Q_s - Q_d \neq 0$  but where there exists an equilibrium price for which  $Q_s - Q_d = 0$ . The existence of such a price does not guarantee the convergence  $Q_s - Q_d \rightarrow 0$ . Even if convergence is expected so that  $Q_s - Q_d \rightarrow 0$ , it can be reached in one of infinitely many combinations of the three variables – supply quantity, demand quantity, and price. An equilibrium theory that does not determine the values of these variables at the limit, or the path by which the limit will be reached, and whether or not the limit will be reached, has little theoretical value and no practical value. This is especially true if equilibrium is proved to exist in economic systems that are incorrectly formulated.

Note also that standard equilibrium theorems apply to a static system while the economic system is dynamic. In addition, while Debreu deals with producers and consumers, the relevant variables are supply and demand, and the theory of these variables is fraught with errors. In addition to the errors mentioned above, consider the following. At best, a producer can estimate the demand for a commodity at various prices, but even if the demand is known precisely at all prices, a producer needs to determine both his supply quantity and its price. With his competitors' quantities and prices unknown, he cannot tell the equilibrium price, in which case he cannot match the equilibrium demand. Furthermore, the supply and demand "curves" of economic theory are not curves and they do not intersect in a point (see Barzilai [1]) and in a com-

petitive economy commodities have multiple prices at the same time so that "the" price of a commodity does not exist.

A related question is whether equilibrium is a desirable state. If in Robinson Crusoe's one-man economy Robinson consumes all that he produces, this economy is in a state of equilibrium. Is being stranded on a remote island an attractive state? Should consumers strive to reach this state? An even more extreme case is the vacuous economy of the moon where total supply equals total demand at zero. Is the equivalent economy of human extinction a state we should wish to reach?

# 5 Summary

Economic theory is founded on errors that require correction. For details see Barzilai [1–12]. In particular, Debreu's *Theory of Value* contributes little to understanding *value* (or *equilibrium*). (Note that while the book is entitled Theory of *Value*, the term *value* does not appear in the book other than in its Preface.)

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