The Challenge of Foundational Errors in Economic and Game Theory*

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> The inertia of the human mind and its resistance to innovation are most clearly demonstrated not, as one might expect, by the ignorant mass – which is easily swayed once its imagination is caught – but by professionals with a vested interest in tradition and in the monopoly of learning. Innovation is a twofold threat to academic mediocrity: it endangers their oracular authority, and it evokes the deeper fear that their whole, laboriously constructed intellectual edifice might collapse.
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> Arthur Koestler, *The Sleepwalkers* (Galileo's Triumph), Hutchinson of London, Page 427, 1968.

1 Mathematical Errors at the Foundations of the Social Sciences

Microeconomics is founded on mathematical errors that have been committed by Pareto and Hicks, and amplified by Samuelson, Debreu [5], von Neumann and Morgenstern, and those who followed them. These persistent and systematic mathematical errors have been propagated or committed independently in game theory, decision theory and, in fact, throughout the social sciences. The errors are logical and technical such as applying mathematical operations where they are not applicable, proving the wrong theorems, applying conclusions where the assumptions that lead to these conclusions are not satisfied, misidentifying the relevant mathematical spaces, using ill-

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defined concepts, misinterpreting the meaning of assumptions (axioms) and conclusions, and more – see Barzilai [1–4] for details.

As a result of these errors, the mathematical models of measurement are incorrect in the classical literature even for the most elementary properties (mass and length); the role of the property under measurement is not understood and, in some important cases, what property is being measured is not understood.

2 Responses to Error Discovery

Responses to the discovery of foundational errors are mostly negative. Some attempt to discredit the discoverer (*he is not an economist* or *the author does not understand modern utility theory*, etc.). Others demonstrate lack of technical competence or understanding of elementary logic and, at the same time, attempt to suppress the message in the hope of preserving their credibility through obfuscation and denial. For example, when glaring errors are exposed, the leaders of INFORMS's Decision Analysis Society proclaim that *there are no errors to be corrected* since *Wakker corrected all that needed correction* and that *Krantz said that the author must be joking*. For a list of "proofs" such as proof by authority, proof by false references, etc., see [6].

Not surprisingly, some leaders in their fields – who fully understand the implications of these errors on their own work and standing – claim that they do not have the time to engage in such "tangential" issues. More surprising are recent referees' claims that although the results are correct and important they should not be published because they are "well known." Implicitly, these referees are claiming that they have been aware of all these errors at the foundations of economic theory, game theory, decision theory, etc., but have chosen to conceal these errors from the rest of us.

Hopefully, by bringing these issues to the attention of the wider community, fewer students will be taught meaningless and incorrect theories.

References

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[6] Paul Renteln and Alan Dundes, "Foolproof: A Sampling of Mathematical Folk Humor," *Notices of the American Mathematical Society,* Vol. 52 No. 1, pp. 28–29, 2005.