

Correcting the Foundations of Operations Research

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References




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- [1] Jonathan Barzilai, “Game Theory Foundational Errors” Parts I–VI.
- [2] Jonathan Barzilai, “Value of a Game,” Notices of the American Mathematical Society, Vol. 55, No. 4, p. 446, April 2008.
- [3] Jonathan Barzilai, “On the Mathematical Foundations of Economic Theory,” pp. 1–13, 2007.


Need for Corrections (1)

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- The chapters on decision theory and game theory in Hillier & Lieberman are fundamentally wrong.
 - Chapters 13, 14, 15 in Winston contain fundamental errors.
 - Decision theory is based on preference measurement. Measurement theory is founded on fundamental errors.

Need for Corrections (2)

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- Decision Analysis Society – no refutation, yet no correction.
 - Howard/Edwards “In Praise of the Old Time Religion”
 - Prescribing the conclusions when the assumptions are not satisfied.

Need for Corrections (3)


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- The leaders of INFORMS's Decision Analysis Society have been aware of foundational errors since at least 2005.
 - Nevertheless, they have taken no action to advise their followers and correct these errors.
 - Decision analysis that is based on flawed methodologies (including utility theory) produces meaningless numbers.

“Foundations of Measurement” has no foundations



- By David H. Krantz, R. Duncan Luce, Patrick Suppes, Amos Tversky, vol. 1 1971, vol 2 1989, vol 3 1990; >1300 pages.
- No definition of “unit”!
- Incorrect models for the most elementary variables – length and weight. (Luce 1992: “Everybody involved in this research has been aware all along that the class of homogeneous structures fails to include a number of scientifically important examples of classical physical measurement and, quite possibly, some that are relevant to the behavioral sciences.”)

Questions

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- On a Scale of 0-10 ... ?
 - Have you seen a proof of the applicability of addition and multiplication on scale values for any scale?
 - Whose values?
 - What's wrong with
 $2008+2009 = 4017?$

The Fundamental Issue




Conditions for applicability of mathematical operations:

- There must be conditions.
- Not in literature.
- (For non-physical variables.)

- Inapplicable operations produce meaningless numbers.

Von Neumann and Morgenstern's Errors

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- Decision theory, game theory, and other theories are founded on errors committed by von Neumann and Morgenstern.
 - Addition and multiplication are not applicable on utility scale values.
 - Additional Game Theory Errors: see “Game Theory Foundational Errors” – Parts I-VI.

Is there a Basis for the Application of Mathematics in the Social Sciences?



Campbell 1920-1940: No!

Von Neumann and Morgenstern:
Yes! Game theory as the mathe-
matical basis for economics and
utility as the basis for game theory.

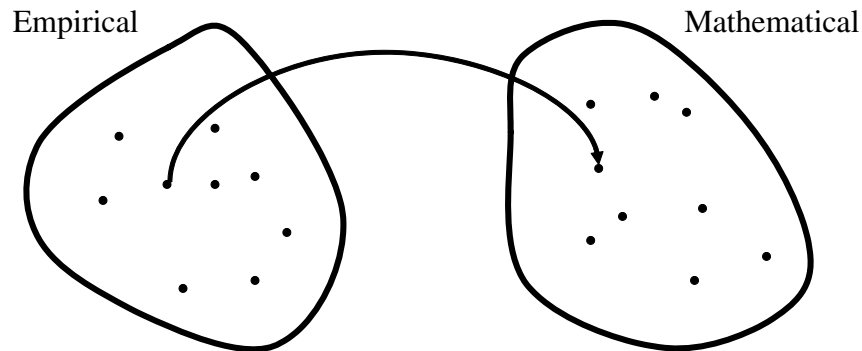
A difficult problem; many errors.

A Sample of Game Theory Errors

- Undefined sums:
 $2008+2009=4017.$
- Values of coalitions: whose values?
- The two-person “zero-sum” game
 $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ with “payoff” table $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$:
Choose $[C, D]$ with probability 0.25 regardless of what D is!

The Modelling Framework

This Framework is *essential!*



- System: set(s) with operations.
- A scale is a mapping that reflects operations (homomorphism).
- Not just any arbitrary mapping.
- The system M is a model of E .
- Property: e.g. adding length.

The Role of the *Property* in Measurement

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- The modelling framework is the only foundation for the mathematical operations.
 - We do not “add objects” - we add their property (length, mass, etc.)!
 - The property is part of the framework.
 - The property of interest here is preference.

The Principle of Reflection




- Mathematical operations are enabled only if they are reflections of corresponding empirical operations.
- Note: order is not an operation.
- Implications: addition and multiplication are not applicable on scale values in the classical literature.

Money is Not a Property of Objects

- Examples of money (Shubik): “coconuts, cocoa beans, dried fish, salt bars, or a beaver pelt” – these are objects, not a property of objects.
- The outcome of measuring temperature is a temperature scale. The outcome of measuring mass is not a length scale.
- Von Neumann and Morgenstern measure preference and produce a utility scale!

Better Understanding is Needed

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- Role of property under measurement.
 - Distinction between objects and properties of objects.
 - Preference, utility, value, etc., are all synonyms.
 - The “utility of value” of an object (Keeney & Raiffa) - the “length of the length” of a pen??!

Applicability of Mathematical Operations, 1887-1940



1887, Helmholtz

1920, Campbell

1932-1940, British Association for the Advancement of Science

1940, Controversy not resolved in Final Report.

Von Neumann and Morgenstern, 1944

The modelling framework has never been in dispute.

Solved the wrong problem:

- Instead of addition and multiplication, axioms for “center of gravity” operation and $t \Rightarrow p + qs$ scale uniqueness.
- But the problem was the empirical operations and $t \Rightarrow p + qs$ does not imply addition and multiplication.

Implications for Economic Theory



- Addition and multiplication are not applicable on ordinal scale values.
- Differentiation is not applicable on ordinal utility.
- If the utility scales of consumer demand theory are ordinal they cannot be differentiated. If they are differentiable they are not ordinal.

Utility Shortcomings



The Principle of Reflection:

- Addition and multiplication are two binary operations; “center of gravity” is one ternary operation.
- Addition and multiplication are not applicable on scale values for *any* scale in the literature including utility scales.

Utility Shortcomings (cont.)




Barzilai's paradox – more important than common utility paradoxes. An intrinsic contradiction:

- Existence and uniqueness vs. construction.
- Framework ok in the abstract. For *preference*, the *interpretation* of the empirical operation leads to a contradiction – prizes unconstrained, lotteries constrained, prizes that are lotteries allowed.

And there is more.

Game Theory

Shortcomings



Game theory values: a fundamental concept. Assigning values to outcomes, coalitions, etc., is constructing value functions.

Value is not a physical variable. Whose values? What is $v(S)$?

The characteristic function of a game and other central game theory concepts including imputations, Shapley's value, and vNM's solution are ill-defined!

Game Theory

Shortcomings (cont.)

Undefined Sums:

$v(S)+v(T)$ is undefined for *utility* scales, *time*, *potential energy*, and similar variables. The sum of “imputations” is undefined. Von Neumann & Morgenstern’s solution is ill-defined.

The utility of a coalition:

Reduction to a two-person game – a coalition vs. its complement. But there is no basis for the utility of a group of players.

Game Theory

Shortcomings (cont.)




“The” value of a game:

The value of a two-person zero-sum game is ill-defined: Utility scales are not unique, $t = p + qs$. Varying p or q changes the value. Any number can be the value of the game.

And there is more.

Back to 1940



Von Neumann and Morgenstern's utility theory and game theory cannot serve as a foundation.

(Note: the notion that the only type of utility that is needed in economic theory is ordinal, is an error.)

The 1940 Final Report



- Missed multiplication.
- Missed the form of addition and multiplication for *time*, *potential energy*, etc.
- Incorrect model even for *mass* and *length*.
- No correction in the literature.

Homogeneous Fields (1)



The problem does have a solution.

The mathematical system M is a field.

The empirical system E is homogeneous.

We need to construct a homogeneous field.

Homogeneous Fields (2)



- A field: one set, two operations including inverse operations. Absolute zero, absolute one.
- A vector space: 2 sets (vectors and scalars), mixed multiplication. Absolute zero.
- A one-dimensional vector space. *Length* and *mass*.
- An affine space: 3 sets (points, vectors, scalars), form of operations.

Homogeneous Fields (3)



- A new classification: weak, proper, and strong scales. Field (scalar), vector, and affine scales.
- Constructing strong affine scales.

Post-vNM

“Measurement Theory”


- The issue is mathematical modeling of measurement, applicability of operations.
- Uniqueness classification: applicability of operations disappears from the literature.
- Addition and multiplication not applicable on scale values for *any* scale in the literature.

“Measurement Theory” (2)




- Model for *length*: addition without multiplication (multiplication is not repeated addition).
- “Extensive” measurement & ratio scales: half the operations are lost; derived from *position*.
- “Difference” measurement & interval scales: the other half is lost; neither addition nor multiplication are applicable.

Ordinal Utility & Indifference Curves

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- Ordinal scale values cannot be differentiated.
 - Differentiable scales are not ordinal.
 - See paper by this title.

AHP

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- Problems mis-diagnosed. Rank reversal is not the main issue. (Dyer's analysis is circular and of no value.)
 - Addition, multiplication, matrix, and vector operations are not applicable; eigenvector is the wrong solution; preference ratios are undefined.
 - Additional errors.